Measuring Holes of 3D Meshes

Yann-Situ Gazull Aldo Gonzalez-Lorenzo Alexandra Bac

March 12, 2021



< ロ > < 同 > < 回 > < 回 >

ъ

Measuring Holes of 3D Meshes Using Medial Axes Prospects Persistent Homology and Hole Measures Medial Axis

▲□▶ ▲圖▶ ▲厘≯ ▲厘≯

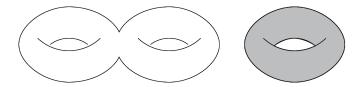
÷.

Introduction

Persistent Homology and Hole Measures Medial Axis

Holes

- 0-holes : connected components
- 1-holes : tunnels
- 2-holes : cavities



two 0-holes

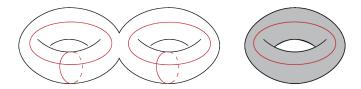
Persistent Homology and Hole Measures Medial Axis

イロト イロト イヨト イヨト

3

Holes

- O-holes : connected components
- 1-holes : tunnels
- 2-holes : cavities



two 0-holes, five 1-holes

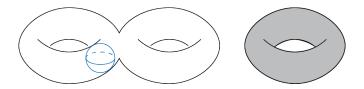
Persistent Homology and Hole Measures Medial Axis

イロン イロン イヨン イヨン

3

Holes

- 0-holes : connected components
- 1-holes : tunnels
- 2-holes : cavities



two 0-holes, five 1-holes and one 2-hole.

Measuring Holes of 3D Meshes Using Medial Axes Prospects Persistent Homology and Hole Measures Medial Axis

4/27

Persistent Homology

Persistent homology keeps track of holes appearing and disappearing in a filtration $(F_t)_{t \in \mathbb{R}}$ as t grows.

Definition (Filtration)

 $(F_t)_{t\in\mathbb{R}}$ is a filtration iff

$$t \leq t' \implies F_t \subset F_{t'}$$

Measuring Holes of 3D Meshes Using Medial Axes Prospects Persistent Homology and Hole Measures Medial Axis

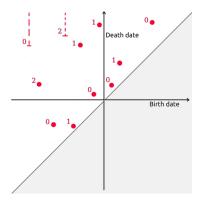
Persistent Homology

Persistent homology keeps track of holes appearing and disappearing in a filtration $(F_t)_{t \in \mathbb{R}}$ as t grows.

Definition (Filtration)

 $(F_t)_{t\in\mathbb{R}}$ is a filtration iff

$$t \leq t' \implies F_t \subset F_{t'}$$



4/27

Persistent Homology and Hole Measures Medial Axis

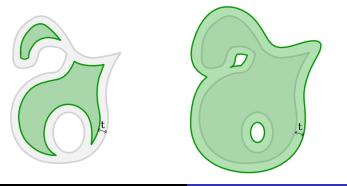
5/27

sdf-Filtration

Definition

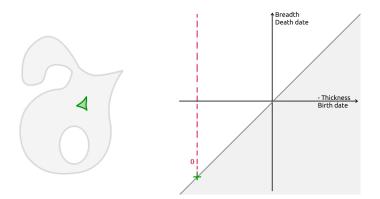
$$\mathcal{F}_t := sdf^{-1}\left(\right] - \infty, t[)$$

 $(\mathcal{F}_t)_{t\in\mathbb{R}}$ is called the *sdf-filtration*.



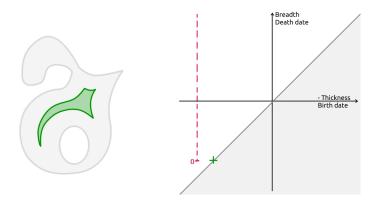
Measuring Holes of 3D Meshes Using Medial Axes Prospects Persistent Homology and Hole Measures Medial Axis

6/27



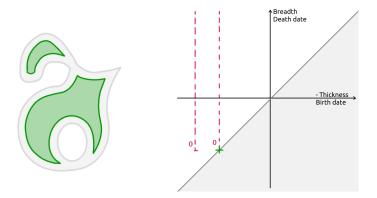
Measuring Holes of 3D Meshes Using Medial Axes Prospects Persistent Homology and Hole Measures Medial Axis

6/27



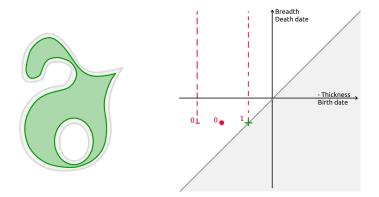
Measuring Holes of 3D Meshes Using Medial Axes Prospects Persistent Homology and Hole Measures Medial Axis

6/27



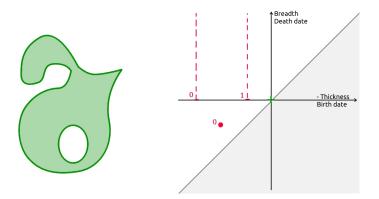
Measuring Holes of 3D Meshes Using Medial Axes Prospects Persistent Homology and Hole Measures Medial Axis

6/27



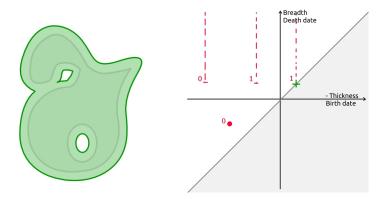
Measuring Holes of 3D Meshes Using Medial Axes Prospects Persistent Homology and Hole Measures Medial Axis

6/27



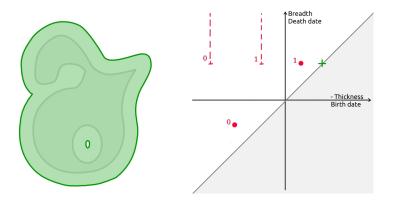
Measuring Holes of 3D Meshes Using Medial Axes Prospects Persistent Homology and Hole Measures Medial Axis

6/27



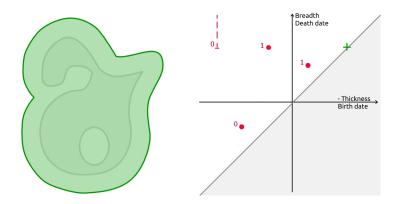
Measuring Holes of 3D Meshes Using Medial Axes Prospects Persistent Homology and Hole Measures Medial Axis

6/27



Measuring Holes of 3D Meshes Using Medial Axes Prospects Persistent Homology and Hole Measures Medial Axis

6/27



Measuring Holes of 3D Meshes Using Medial Axes Prospects

TB-balls

Persistent Homology and Hole Measures Medial Axis

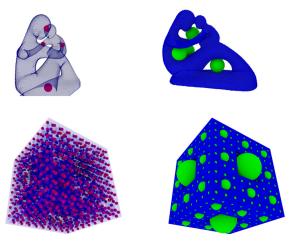
Breadth Death date 01 - Thickness 0

Measuring Holes of 3D Meshes Using Medial Axes Prospects Persistent Homology and Hole Measures Medial Axis

э

8/27

Hole Measures on Cubical Complexes



A. Gonzalez-Lorenzo et al (2016)

Persistent Homology and Hole Measures Medial Axis

▲□▶ ▲圖▶ ▲厘≯ ▲厘≯

÷.

9/27

Medial Axis

Persistent Homology and Hole Measures Medial Axis

The Medial Axis

Definition (Medial Axis)

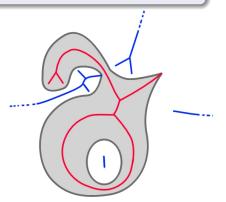
The medial axis $\mathcal{M}(X)$ of X is the set of points that have two or more closest points on the boundary of X.

The inner medial axis :

$$\check{\mathcal{M}}(X) = \mathcal{M}(X) \cap X$$

The outer medial axis :

$$\hat{\mathcal{M}}(X) = \check{\mathcal{M}}(\mathbb{R}^n \backslash X)$$



Persistent Homology and Hole Measures Medial Axis

イロト 不得 とくき とくき とうせい

Properties

Theorem (Lieutier (2003))

For all bounded open X:

 $\check{\mathcal{M}}(X)\approx X$

Where \approx stands for homotopy equivalence.

T-balls from the inner medial axis B-balls from the outer medial axis

・ロト ・ 日 ・ ・ ヨ ・ ・ ヨ ・ ・

э.

Measuring Holes of 3D Meshes Using Medial Axes

T-balls from the inner medial axis B-balls from the outer medial axis

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □

Overview

- Persistence of X on] −∞,0] (and T-balls) can be obtained from persistence of *M*(X).
- Persistence of X on [0, +∞[(and B-ball) can be deduced from persistence of Â(X) mapped to the sphere Sⁿ. [Conjecture]

◆□▶ ◆□▶ ★ 臣▶ ★ 臣▶ 三臣 - のへで

T-balls from the inner medial axis

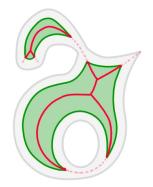
T-balls from the inner medial axis *B*-balls from the outer medial axis

Theorems

Theorem

Given X an open bounded set of \mathbb{R}^n and $t \ge 0$:

$$\mathcal{F}_{-t} \cap \check{\mathcal{M}}(X) = \check{\mathcal{M}}\Big(\mathcal{F}_{-t}\Big)$$



<ロ> (四) (四) (三) (三) (三) (三)

T-balls from the inner medial axis *B*-balls from the outer medial axis

Theorems

Theorem

Given X an open bounded set of \mathbb{R}^n and $t \ge 0$:

$$egin{aligned} \mathcal{F}_{-t} \cap \check{\mathcal{M}}(X) &= \check{\mathcal{M}}\Big(\mathcal{F}_{-t}\Big) \ \mathcal{F}_{-t} \cap \check{\mathcal{M}}(X) &pprox \mathcal{F}_{-t} \end{aligned}$$



T-balls from the inner medial axis *B*-balls from the outer medial axis

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 善臣 - のへで

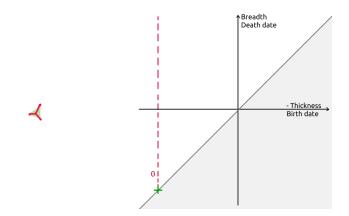
16/27

Theorems

Theorem Given X an open bounded set of \mathbb{R}^n and its associated \mathcal{F}_t : $\mathcal{D}\left((\mathcal{F}_t)_{t\in]-\infty,0]}\right) = \mathcal{D}\left(\left(\mathcal{F}_t \cap \check{\mathcal{M}}(X)\right)_{t\in]-\infty,0]}\right)$

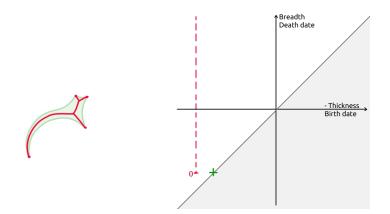
T-balls from the inner medial axis *B*-balls from the outer medial axis

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 善臣 - のへで



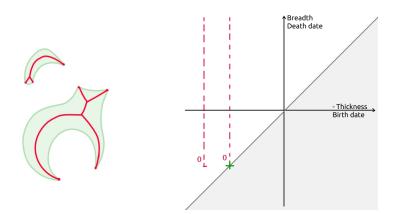
T-balls from the inner medial axis *B*-balls from the outer medial axis

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 善臣 - のへで



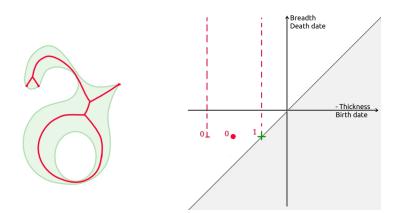
T-balls from the inner medial axis *B*-balls from the outer medial axis

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 善臣 - のへで



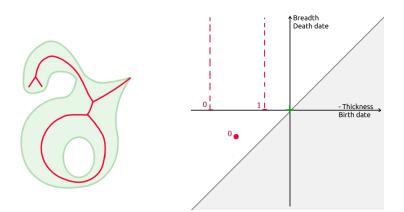
T-balls from the inner medial axis *B*-balls from the outer medial axis

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 善臣 - のへで



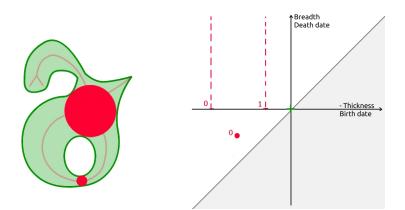
T-balls from the inner medial axis *B*-balls from the outer medial axis

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 善臣 - のへで



T-balls from the inner medial axis *B*-balls from the outer medial axis

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 善臣 - のへで



T-balls from the inner medial axis B-balls from the outer medial axis

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 善臣 - のへで

18/27

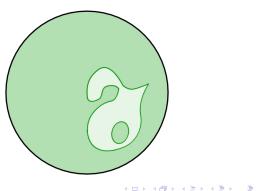
B-balls from the outer medial axis

 $\mathcal{T}\text{-}\mathsf{balls}$ from the inner medial axis $B\text{-}\mathsf{balls}$ from the outer medial axis

Alexander Duality

Theorem (Alexander Duality)

Each *i*-hole in X corresponds to a n - i - 1-hole in $S^n \setminus X$, except for a 0-hole, which corresponds to a 0-hole in $S^n \setminus X$.



T-balls from the inner medial axis *B*-balls from the outer medial axis

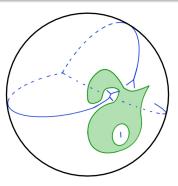
・ロト ・ 日 ・ ・ 日 ・ ・ 日 ・ ・

э.

Conjectures

Main Conjecture

Persistence of X on $[0, +\infty[$ can be deduced from persistence of its outer medial axis $\hat{\mathcal{M}}(X)$ mapped to S^n .



イロト 不得 とうき とうとう ほ

Conjectures

Main Conjecture

Persistence of X on $[0, +\infty[$ can be deduced from persistence of its outer medial axis $\hat{\mathcal{M}}(X)$ mapped to S^n .

Persistence of X on [0, +∞[can be deduced from persistence of Sⁿ\X on] −∞,0] using Alexander duality.



◆□▶ ◆圖▶ ◆臣▶ ◆臣▶ ─ 臣

Conjectures

Main Conjecture

Persistence of X on $[0, +\infty[$ can be deduced from persistence of its outer medial axis $\hat{\mathcal{M}}(X)$ mapped to S^n .

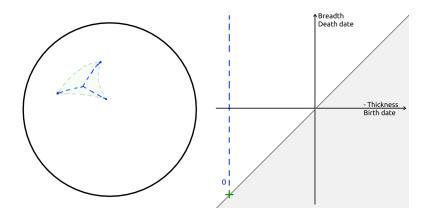
Persistence of X on [0, +∞[can be deduced from persistence of Sⁿ\X on] −∞,0] using Alexander duality.



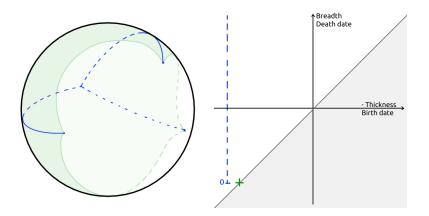
• Persistence of $S^n \setminus X$ on $] - \infty, 0]$ can be obtained from persistence of $\hat{\mathcal{M}}(X) = \check{\mathcal{M}}(S^n \setminus X)$ mapped to S^n .

T-balls from the inner medial axis B-balls from the outer medial axis

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 善臣 - のへで

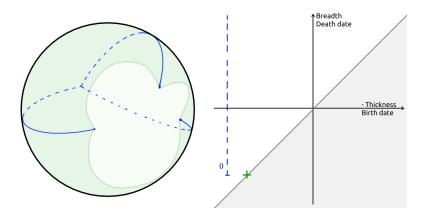


T-balls from the inner medial axis B-balls from the outer medial axis



T-balls from the inner medial axis B-balls from the outer medial axis

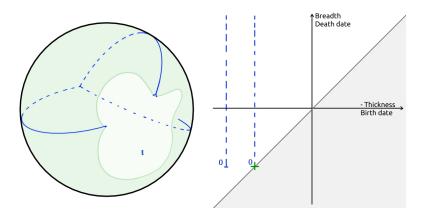
◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 善臣 - のへで



T-balls from the inner medial axis B-balls from the outer medial axis

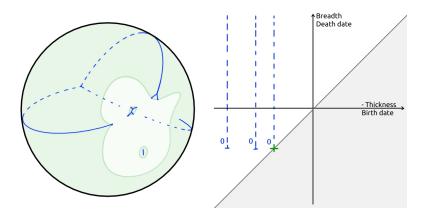
◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 善臣 - のへで

Complementary Holes



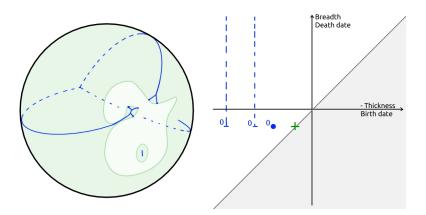
Yann-Situ Gazull, A. Gonzalez-Lorenzo, A. Bac Measuring Holes of 3D Meshes

T-balls from the inner medial axis B-balls from the outer medial axis



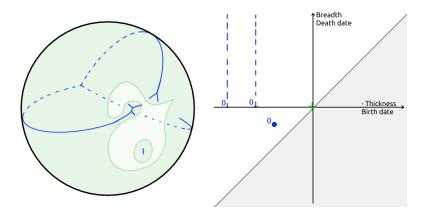
T-balls from the inner medial axis B-balls from the outer medial axis

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 善臣 - のへで



T-balls from the inner medial axis B-balls from the outer medial axis

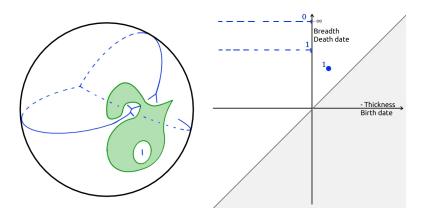
◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 善臣 - のへで



T-balls from the inner medial axis B-balls from the outer medial axis

◆□ ▶ ◆□ ▶ ◆ □ ▶ ◆ □ ▶ ● □ ■

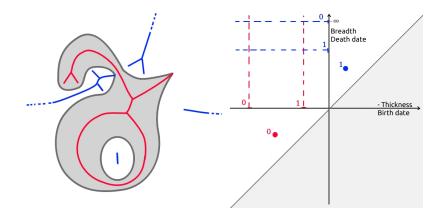
Alexander Deduction



T-balls from the inner medial axis B-balls from the outer medial axis

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 善臣 - のへで

Alexander Deduction

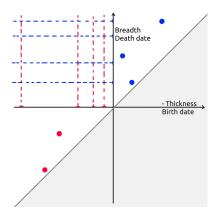


T-balls from the inner medial axis B-balls from the outer medial axis

・ロト ・四ト ・ヨト ・ヨト

æ -

Partial Persistence



Yann-Situ Gazull, A. Gonzalez-Lorenzo, A. Bac Measuring Holes of 3D Meshes

Medial Axis Computation Complete Persistence

・ロト ・ 日 ・ ・ 日 ・ ・ 日 ・

Ξ.

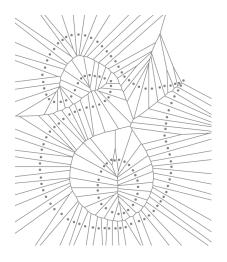
Prospects

Yann-Situ Gazull, A. Gonzalez-Lorenzo, A. Bac Measuring Holes of 3D Meshes

Medial Axis Computation Complete Persistence

Medial Axes using Voronoï Diagrams

- Giensen (2011)
- Cazals (2008)
- K.Dey (2004)



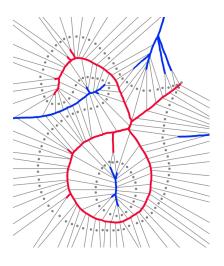
・ロト ・聞 ト ・ ヨ ト ・ ヨ ト

э

Medial Axis Computation Complete Persistence

Medial Axes using Voronoï Diagrams

- Giensen (2011)
- Cazals (2008)
- K.Dey (2004)



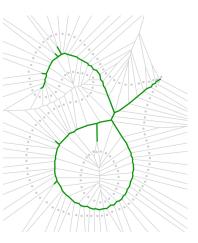
< ロ > < 同 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < 回 > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ >

э

Medial Axis Computation Complete Persistence

Full Persistence using Voronoï Filtration





・ロト ・ 日 ・ ・ ヨ ・ ・ ヨ ・ ・

э

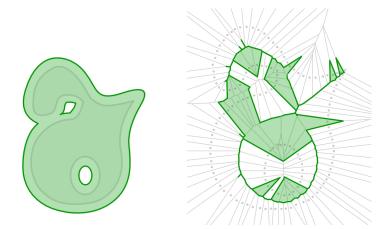
26/27

Medial Axis Computation Complete Persistence

э

26/27

Full Persistence using Voronoï Filtration

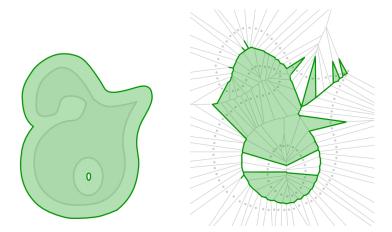


Medial Axis Computation Complete Persistence

э

26/27

Full Persistence using Voronoï Filtration



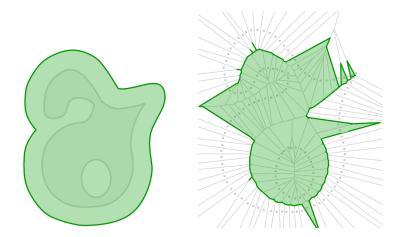
Medial Axis Computation Complete Persistence

◆□ > ◆□ > ◆臣 > ◆臣 > ○

э

26/27

Full Persistence using Voronoï Filtration



- Every hole has two independent measures that can be represented using balls.
- Persistence on medial axes of X provide partial persistence and every TB-ball of X.

<□> <□> <□> <□> <=> <=> <=> <=> ○QC 27/27